



Oxford Cambridge and RSA

**Monday 13 May 2019 – Afternoon**

**AS Level Further Mathematics B (MEI)**

**Y410/01 Core Pure**

**Time allowed: 1 hour 15 minutes**



**You must have:**

- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

**You may use:**

- a scientific or graphical calculator

**INSTRUCTIONS**

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION**

- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets [ ].
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions

**1 In this question you must show detailed reasoning.**

Find  $\sum_{r=1}^{100} \left( \frac{1}{r} - \frac{1}{r+2} \right)$ , giving your answer correct to 4 decimal places. [3]

**2** The roots of the equation  $3x^2 - x + 2 = 0$  are  $\alpha$  and  $\beta$ .  
Find a quadratic equation with integer coefficients whose roots are  $2\alpha - 3$  and  $2\beta - 3$ . [3]

**3 In this question you must show detailed reasoning.**

**A** and **B** are matrices such that  $\mathbf{B}^{-1}\mathbf{A}^{-1} = \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix}$ .

(a) Find **AB**. [3]

(b) Given that  $\mathbf{A} = \begin{pmatrix} \frac{1}{3} & 1 \\ 0 & 1 \end{pmatrix}$ , find **B**. [3]

**4** (a) Find  $\mathbf{M}^{-1}$ , where  $\mathbf{M} = \begin{pmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ -2 & 1 & 2 \end{pmatrix}$ . [1]

(b) Hence find, in terms of the constant  $k$ , the point of intersection of the planes

$$\begin{aligned} x + 2y + 3z &= 19, \\ -x + y + 2z &= 4, \\ -2x + y + 2z &= k. \end{aligned} \quad [3]$$

(c) **In this question you must show detailed reasoning.**

Find the acute angle between the planes  $x + 2y + 3z = 19$  and  $-x + y + 2z = 4$ . [4]

**5** Prove by induction that, for all positive integers  $n$ ,  $\sum_{r=1}^n \frac{1}{3^r} = \frac{1}{2} \left( 1 - \frac{1}{3^n} \right)$ . [6]

- 6 A linear transformation  $T$  of the  $x$ - $y$  plane has an associated matrix  $\mathbf{M}$ , where  $\mathbf{M} = \begin{pmatrix} \lambda & k \\ 1 & \lambda - k \end{pmatrix}$ , and  $\lambda$  and  $k$  are real constants.

(a) You are given that  $\det \mathbf{M} > 0$  for all values of  $\lambda$ .

(i) Find the range of possible values of  $k$ . [3]

(ii) What is the significance of the condition  $\det \mathbf{M} > 0$  for the transformation  $T$ ? [1]

For the remainder of this question, take  $k = -2$ .

(b) Determine whether there are any lines through the origin that are invariant lines for the transformation  $T$ . [4]

(c) The transformation  $T$  is applied to a triangle with area 3 units<sup>2</sup>. The area of the resulting image triangle is 15 units<sup>2</sup>.  
Find the possible values of  $\lambda$ . [3]

7 (a) Sketch on a single Argand diagram

(i) the set of points for which  $|z - 1 - 3i| = 3$ , [3]

(ii) the set of points for which  $\arg(z + 4) = \frac{1}{4}\pi$ . [3]

(b) Find, in exact form, the two values of  $z$  for which  $|z - 1 - 3i| = 3$  and  $\arg(z + 4) = \frac{1}{4}\pi$ . [6]

8 **In this question you must show detailed reasoning.**

You are given that  $i$  is a root of the equation  $z^4 - 2z^3 + 3z^2 + az + b = 0$ , where  $a$  and  $b$  are real constants.

(a) Show that  $a = -2$  and  $b = 2$ . [4]

(b) Find the other roots of this equation. [7]

**END OF QUESTION PAPER**

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